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## DYNAMICS OF INFLATIONARY COSMOLOGY IN TVSD MODEL

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Within the framework of a model Universe with time variable space dimensions (TVSD), known as decrumpling or TVSD model, we study TVSD chaotic inflation and obtain dynamics of the inflaton, scale factor and spatial dimension. We also study the quantum fluctuations of the inflaton field and obtain the spectral index and its running in this model. Two classes of examples have been studied and comparisons made with the standard slow-roll formulae. We compare our results with the recent Wilkinson Microwave Anisotropy Probe (WMAP) data.

*Keywords:* Inflation; extra spatial dimensions; quantum fluctuations.

## 1. Introduction

One of the most intriguing challenges in modern physics is to find observable consequences of different kinds of theories in higher dimensions. We here present an inflationary model, known as decrumpling inflation model in which the number of spatial dimension has a dynamical behavior and decreases during the expansion of the Universe with a rate to be less than about  $10^{-14}\text{yr}^{-1}$ .<sup>1</sup>

Our motivation to study decrumpling inflation is to investigate cosmological implications of time variability of the number of spatial dimensions. To do so, we compute the spectral index and its running within the framework of decrumpling inflation. For more details about a model Universe with time variable space dimensions (TVSD), known as TVSD or decrumpling model, see Refs. [1-6].

To do this research an important conceptual issue is properly dealt with the meaning of time variability of the number of spatial dimensions. In Ref. [5], this conceptual issue has been discussed in detail.

Although time variability of spatial dimensions have not been firmly achieved in experiments and theories, such dynamical behavior of the spatial dimensions should not be ruled out in the context of cosmology and astroparticle physics.

Here, we will be concerned with the approaches proposed in the pioneer paper<sup>12</sup> where the cosmic expansion of the Universe is named decrumpling expansion and is due to decrease of the number of spatial dimensions.

The most important difference between decrumpling model and other attempts about the time evolution of spatial dimension is that in this model the number of extra spatial dimensions changes with time while in other theories the size of extra spatial dimensions is a dynamical parameter. Based on time variability of the size of spatial dimensions it has been reported<sup>1</sup> that the present rate of change of the mean radius of any additional spatial dimensions to be less than about  $10^{-19}\text{yr}^{-1}$ . It is worth mentioning that this result is based on dynamical behavior of the size of extra spatial dimensions while in decrumpling model we take the size of extra spatial dimensions to be constant and the number of spatial dimensions decreases continuously as the Universe expands. The present rate of time variation of the number of the spatial dimensions in decrumpling or TVSD model is about  $10^{-13}\text{yr}^{-1}$ .<sup>1</sup>

Another subject which lately has attracted much attention is the scalar spectral index and its running.

Primordial perturbations from inflation currently provide our only complete model for the generation of structure in the Universe. It is commonly stated that a generic prediction of inflationary models is a scale-invariant spectrum of adiabatic perturbations, characterized by a scalar spectral index  $n_S$  that obeys  $n_S - 1 = 0$ . However, this statement is only true for very special spacetimes like a pure de Sitter spacetime, which does not describe our cosmological history. For nearly all realistic inflationary models, the value of  $n_S$  will vary with the wave number  $k$ .

Typically, since  $n_S - 1 \simeq 0$  on the scales probed by the cosmic microwave background (CMB), the deviations from a constant  $n_S$  must be small. Nevertheless, increasingly accurate cosmological observations provide information about the scalar spectral index on scales below those accessible to anisotropy measurements of the CMB. On such a wide range of scales, it is entirely possible that  $n_S$  will exhibit significant running, a value that depends on the scale on which it is measured. Such running is quantified by the derivative  $dn_S/d\ln k$  and, in fact, recently released data from the WMAP satellite indicates that<sup>7</sup>

$$n_S(k_0 = 0.05 \text{ Mpc}^{-1}) = 0.93^{+0.03}_{-0.03}, \quad (1)$$

$$\frac{dn_S}{d\ln k}(k_0 = 0.05 \text{ Mpc}^{-1}) = -0.031^{+0.016}_{-0.018}. \quad (2)$$

The limits on the  $n_S$  and its running using WMAP data alone are<sup>8</sup>

$$n_S(k_0 = 0.002 \text{ Mpc}^{-1}) = 1.20^{+0.12}_{-0.11}, \quad (3)$$

$$\frac{dn_S}{d\ln k}(k_0 = 0.002 \text{ Mpc}^{-1}) = -0.077^{+0.050}_{-0.052}, \quad (4)$$

where  $k_0$  is some pivot wave number.

Recent data, including that from the WMAP data satellite, show some evidence that the index runs (changes as a function of the scale  $k$  at which it is measured) from  $n_S > 1$  (blue) on long scales to  $n_S < 1$  (red) on short scales. The authors of Ref.[9] investigated the extent to which inflationary models can accommodate such significant running of  $n_S$ . They show that within the slow-roll approximation, the fact that  $n_S - 1$  changes sign from blue to red forces the slope of the potential to reach a minimum at a similar field location.

Chaotic inflation models are usually based upon potentials of the form  $V(\phi) = a\phi^b$  ( $a$  is a constant and  $b = 2, 4, \dots$  is an even integer). For the interesting cases of  $b = 2$  and 4, it has been shown that<sup>10</sup>

$$\frac{dn_S}{d \ln k} = -0.8 \times 10^{-3} \quad (b = 2, \mathcal{N} = 50), \quad (5)$$

$$\frac{dn_S}{d \ln k} = -1.2 \times 10^{-3} \quad (b = 4, \mathcal{N} = 50), \quad (6)$$

where  $\mathcal{N}$  is the e-folding number.

We will use the natural units system that sets  $k_B$ ,  $c$ , and  $\hbar$  all equal to one, so that  $\ell_P = M_P^{-1} = \sqrt{G}$ . To read easily this article we also use the notation  $D_t$  instead of  $D(t)$  that means the space dimension  $D$  is as a function of time.

The plan of this article is as follows. In section 2, we give a brief review of decrumpling or TVSD model. In section 3, we present the dynamical solutions of  $\phi(t)$ ,  $a(t)$  and  $D_t(t)$ . In section 4, we first present the explicit and general formulae for the spectral index and its running within the framework of decrumpling or TVSD inflation and then apply them to two classes of examples of the inflaton potential. Finally, we discuss our results and conclude in section 5.

## 2. Review of Decrumpling or TVSD Model

Decrumpling model is based on the assumption that the basic blocks of the space-time are fractaly structured.<sup>5,12</sup> In the pioneer paper<sup>12</sup> the spatial dimension of the Universe was considered as a continuous time dependent variable. As the Universe expands, its spatial dimension decreases continuously, thereby generating what has been named a decrumpling Universe. Then this model has been overlooked and the quantum cosmological aspects, as well as, a possible test theory for studying time evolution of Newton's constant have also been discussed.<sup>1,6</sup> Chaotic inflation in decrumpling model and its dynamical solutions have also been studied.<sup>2,3,4</sup>

The concept of decrumpling expansion of the Universe is inspired by the idea of decrumpling coming from polymer physics.<sup>5,12</sup> In this model the fundamental building blocks of the Universe are like cells with arbitrary dimensions having in each dimension a characteristic size  $\delta$  which maybe of the order of the Planck length  $\mathcal{O}(10^{-33} \text{ cm})$  or even smaller so that the minimum physical radius of the Universe is  $\delta$ . These "space cells" are embedded in a  $\mathcal{D}$  space, where  $\mathcal{D}$  may be up to infinity. Therefore, the space dimensions of the Universe depend on how these fundamental

cells are configured in this embedding space. The Universe may have begun from a very crumpled state having a very high dimension  $\mathcal{D}$  and a size  $\delta$ , then have lost dimension through a uniform decrumpling which we see like a uniform expansion. The expansion of space, being now understood like a decrumpling of cosmic space, reduces the space-time dimension continuously from  $\mathcal{D} + 1$  to the present value  $D_0 + 1$ . In this picture, the Universe can have any space dimension. As it expands, the number of spatial dimensions decreases continuously. The physical process that causes or necessitates such a decrease in the number of spatial dimensions comes from how these fundamental cells are embedded in a  $\mathcal{D}$  space.

As an example, take a limited number of small three-dimensional beads. Depending on how these beads are embedded in space they can configure to a one-dimensional string, two-dimensional sheet, or three-dimensional sphere. This is the picture we are familiar with from the concept of crumpling in polymer physics where a crumpled polymer has a dimension more than one. Or take the picture of a clay which can be like a three-dimensional sphere, or a two-dimensional sheet, or even a one-dimensional string, a picture based on the theory of fluid membranes.

While it is common to make *ad hoc* assumptions in cosmological model building in the absence of a complete theory of quantum gravity, some of the particular ingredients of the model owe their physical basis perhaps more to polymer physics than to cosmology. Progress with decrumpling model can only be made if there is a breakthrough in terms of finding a natural mechanism for varying the number of spatial dimensions in some alternative fashion to that which is considered here.<sup>1</sup>

### 2.1. Relation between the effective space dimension $D_t(t)$ and characteristic size of the Universe $a(t)$

Assume the Universe consists of a fixed number  $\mathbb{N}$  of universal cells having a characteristic length  $\delta$  in each of their dimensions. The volume of the Universe at the time  $t$  depends on the configuration of the cells. It is easily seen that<sup>5</sup>

$$\text{vol}_{D_t}(\text{cell}) = \text{vol}_{D_0}(\text{cell})\delta^{D_t-D_0}, \quad (7)$$

where the  $t$  subscript in  $D_t$  means that  $D$  to be as a function of time, i.e.  $D(t)$ .

Interpreting the radius of the Universe,  $a$ , as the radius of gyration of a crumpled “universal surface”, the volume of space can be written<sup>5</sup>

$$\begin{aligned} a^{D_t} &= \mathbb{N} \text{vol}_{D_t}(\text{cell}) \\ &= \mathbb{N} \text{vol}_{D_0}(\text{cell})\delta^{D_t-D_0} \\ &= a_0^{D_0}\delta^{D_t-D_0} \end{aligned} \quad (8)$$

or

$$\left(\frac{a}{\delta}\right)^{D_t} = \left(\frac{a_0}{\delta}\right)^{D_0} = e^C, \quad (9)$$

where  $C$  is a universal positive constant. Its value has a strong influence on the dynamics of spacetime, for example on the dimension of space, say, at the Planck

time. Hence, it has physical and cosmological consequences and may be determined by observation. The zero subscript in any quantity, e.g. in  $a_0$  and  $D_0$ , denotes its present value. We coin the above relation as a “dimensional constraint” which relates the “scale factor” of decrumpling model to the spatial dimension. We consider the comoving length of the Hubble radius at present time to be equal to one. So the interpretation of the scale factor as a physical length is valid. The dimensional constraint can be written in this form<sup>5</sup>

$$\frac{1}{D_t} = \frac{1}{C} \ln \left( \frac{a}{a_0} \right) + \frac{1}{D_0}. \quad (10)$$

It is seen that by the expansion of the Universe, the space dimension decreases. Time derivative of (9) or (10) leads to

$$\dot{D}_t = -\frac{D_t^2 \dot{a}}{C a}. \quad (11)$$

It can be easily shown that the case of constant space dimension corresponds to when  $C$  tends to infinity. In other words,  $C$  depends on the number of fundamental cells. For  $C \rightarrow +\infty$ , the number of cells tends to infinity and  $\delta \rightarrow 0$ . In this limit, the dependence between the space dimensions and the radius of the Universe is removed, and consequently we have a constant space dimension.<sup>5</sup>

## 2.2. Physical meaning of $D_P$

We define  $D_P$  as the space dimension of the Universe when the scale factor is equal to the Planck length  $\ell_P$ . Taking  $D_0 = 3$  and the scale of the Universe today to be the present value of the Hubble radius  $H_0^{-1}$  and the space dimension at the Planck length to be 4, 10, or 25, from Kaluza-Klein and superstring theory, we can obtain from (9) and (10) the corresponding value of  $C$  and  $\delta$

$$\frac{1}{D_P} = \frac{1}{C} \ln \left( \frac{\ell_P}{a_0} \right) + \frac{1}{D_0} = \frac{1}{C} \ln \left( \frac{\ell_P}{H_0^{-1}} \right) + \frac{1}{3}, \quad (12)$$

$$\delta = a_0 e^{-C/D_0} = H_0^{-1} e^{-C/3}. \quad (13)$$

In Table 1, values of  $C$ ,  $\delta$  and also  $\dot{D}_t|_0$  for some interesting values of  $D_P$  are given. These values are calculated by assuming  $D_0 = 3$  and  $H_0^{-1} = 3000 h_0^{-1} \text{Mpc} = 9.2503 \times 10^{27} h_0^{-1} \text{cm}$ , where we take  $h_0 = 1$ .<sup>5</sup>

### 3. Dynamical Solutions of $\phi(t)$ , $a(t)$ , and $D_t(t)$

Using the slow-roll approximation, equations of motion of  $a(t)$ ,  $D_t(t)$ , and  $\phi(t)$  for the potential  $V(\phi) = m^2\phi^2/2$  are given by<sup>3</sup>

$$\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{8\pi m^2\phi^2}{D_t(D_t-1)M_P^2}, \quad (14)$$

$$\frac{D_t^2 \dot{a} \dot{\phi}}{a} \left[ \frac{1}{D_0} - \frac{1}{C} \left\{ \ln \chi_c + \frac{1}{2} \ln \pi - \frac{1}{2} \psi \left( \frac{D_t}{2} + 1 \right) \right\} \right] \simeq -m^2\phi, \quad (15)$$

$$\dot{D}_t^2 \simeq \frac{8\pi D_t^3 m^2 \phi^2}{C^2(D_t-1)M_P^2}, \quad (16)$$

where  $m$  is the inflaton mass,  $m = 1.21 \times 10^{-6} M_P$ . In natural unit system  $M_P = 2.18 \times 10^{-5} \text{gr}$  and  $1 \text{gr} = 8.52 \times 10^{47} \text{sec}^{-1}$ . So the Planck mass is  $M_P = 1.85 \times 10^{43} \text{sec}^{-1}$  and the inflaton mass is  $m = 2.24 \times 10^{37} \text{sec}^{-1}$ .<sup>13</sup>

In [3] we obtained dynamical solutions of Eqs.(14-16). Using definition of the following parameters<sup>3</sup>

$$\alpha \equiv -\frac{mD_0}{2D_i} \sqrt{\frac{D_i-1}{2\pi D_i}}, \quad (17)$$

$$\beta \equiv \frac{m(2D_i-3)}{(D_i-1)} \sqrt{\frac{\pi D_i}{2(D_i-1)}}, \quad (18)$$

and

$$\gamma \equiv -\alpha + \frac{\beta}{C} \left( \frac{\phi_i}{M_P} \right)^2, \quad (19)$$

the solutions of Eqs.(14-16) are given by<sup>3</sup>

$$\frac{\phi(t)}{M_P} = \sqrt{\frac{\gamma C}{\beta}} \tanh \left[ -\sqrt{\frac{\beta \gamma}{C}}(t-t_i) + \text{arctanh} \left( \frac{\phi_i}{M_P} \sqrt{\frac{\beta}{C\gamma}} \right) \right] + \dots, \quad (20)$$

$$\begin{aligned} D_t(t) = & \left( 1 + \left\{ 1 - \frac{2}{D_i} \left( 1 - \frac{1}{2D_i} \right) - \frac{8\pi}{CD_0} \left[ \left( \frac{\phi_i}{M_P} \right)^2 \right. \right. \right. \\ & \left. \left. - \frac{\gamma C}{\beta} \tanh^2 \left[ -\sqrt{\frac{\beta \gamma}{C}}(t-t_i) + \text{arctanh} \left( \frac{\phi_i}{M_P} \sqrt{\frac{\beta}{C\gamma}} \right) \right] \right] \right\}^{1/2} \right) \\ & \times \left\{ \frac{2}{D_i} \left( 1 - \frac{1}{2D_i} \right) + \frac{8\pi}{CD_0} \left[ \left( \frac{\phi_i}{M_P} \right)^2 - \frac{\gamma C}{\beta} \tanh^2 \left[ -\sqrt{\frac{\beta \gamma}{C}}(t-t_i) \right. \right. \right. \\ & \left. \left. \left. + \text{arctanh} \left( \frac{\phi_i}{M_P} \sqrt{\frac{\beta}{C\gamma}} \right) \right] \right] \right\}^{-1}, \end{aligned} \quad (21)$$

$$a(t) = a_i \exp \left( \frac{8\pi D_t [\phi_i^2 - \phi^2(t)]}{D_0 M_P^2 [(2 - \frac{1}{D_i}) D_t - 1]} \right). \quad (22)$$

Table 1. Values of  $D_P$ ,  $C$ ,  $\delta$ ,  $\dot{D}_t|_0$ ,  $D_i$ ,  $\phi_i/M_P$ ,  $\alpha$ ,  $\beta$ , and  $\gamma$ .  $D_P = 3, 4$  and  $10$  are corresponding to  $a_i = 1.02 \times 10^{-25}$ ,  $1.06 \times 10^{-30}$ , and  $2.28 \times 10^{-43}$  cm, respectively.

$D_P$	$C$	$\delta(\text{cm})$	$\dot{D}_t _0(\text{yr}^{-1})$	$D_i$	$\phi_i/M_P$	$\alpha$	$\beta$	$\gamma$
3	$+\infty$	0	0	3.00	3.060	$-0.16m$	$2.30m$	$0.16m$
4	1678.8	$8.6 \times 10^{-216}$	$-5.48 \times 10^{-13}$	3.94	3.491	$-0.13m$	$2.41m$	$0.15m$
10	599.57	$1.5 \times 10^{-59}$	$-1.53 \times 10^{-12}$	16.08	4.512	$-0.04m$	$2.50m$	$0.12m$

The  $i$  subscript in  $D_i$ ,  $\phi_i$  and  $a_i$  are their values at the beginning of inflationary epoch. Their values are given in Ref. [3], see Table 1. Inserting the values of  $D_0 = 3$ ,  $\phi_i$  and  $D_i$  in equations (17), (18) and (19), one can easily obtain the values of  $\alpha$ ,  $\beta$  and  $\gamma$ , see Table 1. When the space dimension at the Planck length to be equal to  $D_P = 4$  corresponding to  $C = 1678.8$ , one can easily obtain the dynamical solutions of  $\phi(t)$ ,  $D_t(t)$  and  $a(t)$ , which are given by

$$\frac{\phi(t)}{M_P} = 10.22 \tanh \left[ -2.24 \times 10^{35}(t - t_i) + \text{arctanh } 0.34 \right], \quad (23)$$

$$D_t(t) = \left( 1 + \left\{ 0.56 - 4.99 \times 10^{-3} \left[ 12.19 - 104.49 \tanh^2 \left[ -2.24 \times 10^{35}(t - t_i) + \text{arctanh } 0.34 \right] \right] \right\}^{1/2} \right) \times \left\{ 0.44 + 4.99 \times 10^{-3} \left[ 12.19 - 104.49 \right. \right. \\ \left. \left. \times \tanh^2 \left[ -2.24 \times 10^{35}(t - t_i) + \text{arctanh } 0.34 \right] \right] \right\}^{-1}, \quad (24)$$

$$a(t) = 1.06 \times 10^{-30} \exp \left[ \frac{8\pi D_t \left( 12.19 - \left( \frac{\phi(t)}{M_P} \right)^2 \right)}{3(1.75 D_t - 1)} \right]. \quad (25)$$

Using Equations (17)-(22), for the space dimension at the Planck length  $D_P$  to be equal to 10 corresponding to  $C = 599.57$ , one can easily obtain the dynamical solutions of  $\phi(t)$ ,  $D_t(t)$  and  $a(t)$ , which are given by

$$\frac{\phi(t)}{M_P} = 5.36 \tanh \left[ -4.48 \times 10^{35}(t - t_i) + \text{arctanh } 0.84 \right], \quad (26)$$

$$D_t(t) = \left( 1 + \left\{ 0.88 - 0.01 \left[ 20.36 - 28.78 \times \tanh^2 \left[ -1.12 \times 10^{34}(t - t_i) + \text{arctanh } 0.84 \right] \right] \right\}^{1/2} \right) \times \left\{ 0.12 + 0.01 \left[ 20.36 - 28.78 \right. \right. \\ \left. \left. \times \tanh^2 \left[ -4.48 \times 10^{35}(t - t_i) + \text{arctanh } 0.84 \right] \right] \right\}^{-1}, \quad (27)$$

$$a(t) = 2.28 \times 10^{-43} \exp \left( \frac{8\pi D_t [20.36 - \left( \frac{\phi(t)}{M_P} \right)^2]}{3[1.94 D_t - 1]} \right). \quad (28)$$

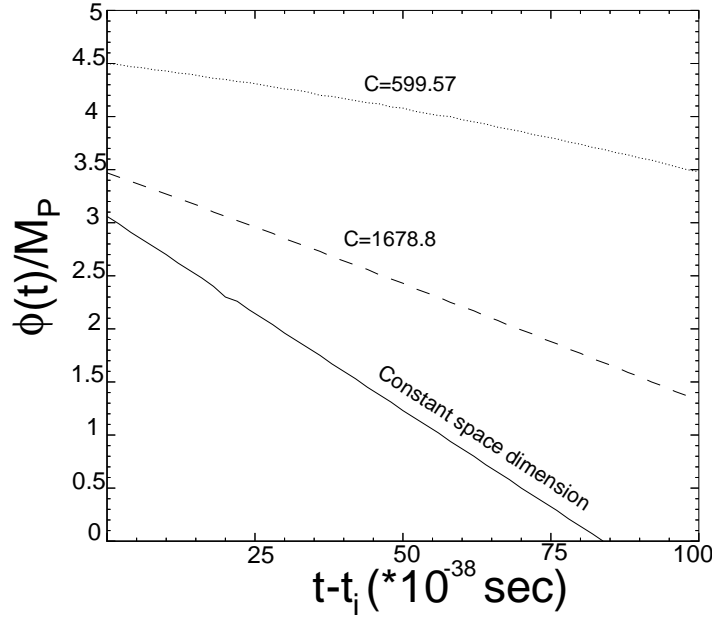


Fig. 1. Inflaton field as a function of time for  $C = 1678.8$  (dashed line),  $C = 599.57$  (dotted line), and three constant space dimension or  $C \rightarrow +\infty$  (solid line).

In three constant space dimension corresponding to  $C \rightarrow +\infty$ , one can use Eqs. (17)-(22) and obtain the following dynamical solutions of  $\phi(t)$  and  $a(t)$  which are given by

$$\frac{\phi(t)}{M_P} = 3.060 - 3.65 \times 10^{36}(t - t_i), \quad (29)$$

$$a(t) = 1.02 \times 10^{-25} \exp \left( 2\pi \left[ 9.36 - \left( \frac{\phi(t)}{M_P} \right)^2 \right] \right). \quad (30)$$

Using Eqs. (23), (26), and (29), we plot the time evolution of the inflaton field in Fig.(1). In Fig.(2), time evolution of the spatial dimension has been shown by Eqs. (24) and (27). Finally in Fig.(3), the time evolution of the scale factor has been shown by Eqs. (25), (28) and (30). It is worth mentioning that we use log scale for the axis of the scale factor in Fig.(3). As mentioned in [3], since we take the comoving length of the Hubble radius to be equal to one, so the scale factor is equal to the physical length. As shown in [3], the inflationary epoch lasts about  $1.003 \times 10^{-36}$  sec and  $1.159 \times 10^{-36}$  sec for  $D_P = 4$  and  $D_P = 10$ , respectively. In



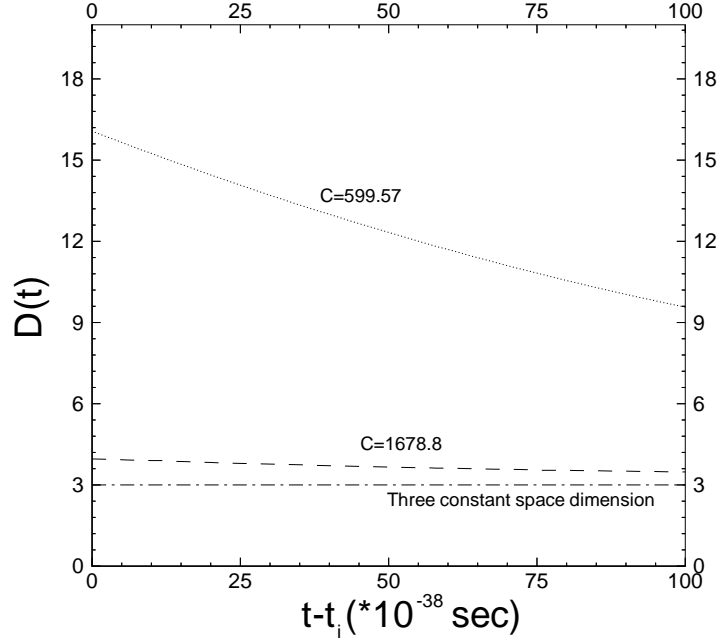


Fig. 2. Space dimensions as a function of time for  $C = 1678.8$  (dashed line),  $C = 599.57$  (dotted line), and three constant space dimension or  $C \rightarrow +\infty$  (dashdot line).

three constant space dimension the inflationary epoch lasts about  $7.598 \times 10^{-37}$  sec.<sup>3</sup> We have used these values of time for plotting Figs.(1), (2) and (3).

#### 4. Running of the Spectral Index in Decrumpling or TVSD Inflation

In this section, we present explicit and general formulae for the spectral index and its running within the framework of TVSD or decrumpling inflation. For the purposes of illustration, we apply our results for two classes of examples of the inflaton potential.

##### 4.1. Decrumpling or TVSD chaotic inflation

Inflation has been studied in the framework of decrumpling or TVSD model. The crucial equations are<sup>3,4</sup>

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{16\pi}{D_t(D_t - 1)M_P^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{k}{a^2}, \text{ Friedmann equation, (31)}$$

$$\ddot{\phi} + D_t H \dot{\phi} + \dot{D}_t \dot{\phi} \left(\ln \frac{a}{a_0} + \frac{d \ln V_{D_t}}{d D_t}\right) = -V'(\phi), \quad \text{Fluid equation, (32)}$$

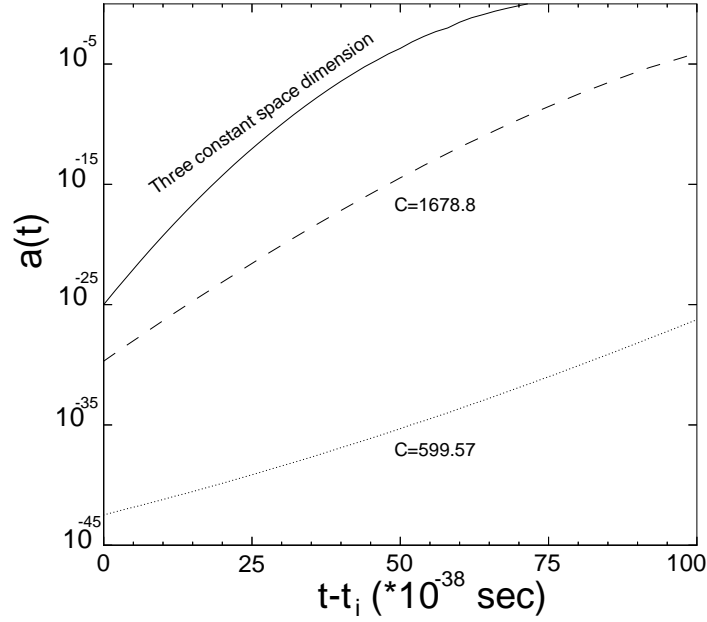


Fig. 3. Scale factor as a function of time for  $C = 1678.8$  (dashed line),  $C = 599.57$  (dotted line), and three-constant space dimension or  $C \rightarrow +\infty$  (solid line). We use the log scale for the axis of  $a(t)$ . Here the scale factor is the same as the physical length because we take the comoving length of the Hubble radius to be equal to one.

where  $V_{D_t}$  is the volume of the space-like sections<sup>5</sup>

$$V_{D_t} = \begin{cases} \frac{2\pi^{(D_t+1)/2}}{\Gamma[(D_t+1)/2]}, & \text{if } k=+1, \text{ closed decrumpling model,} \\ \frac{\pi^{(D_t/2)}}{\Gamma(D_t/2+1)} \chi_c^{D_t}, & \text{if } k=0, \text{ flat decrumpling model,} \\ \frac{2\pi^{(D_t/2)}}{\Gamma(D_t/2)} f(\chi_c), & \text{if } k=-1, \text{ open decrumpling model.} \end{cases} \quad (33)$$

These volumes of space-like sections are valid even in the case of constant  $D$ -space.<sup>5</sup> Here  $\chi_c$  is a cut-off and  $f(\chi_c)$  is a function thereof.<sup>2</sup>

Using the slow-roll approximation in decrumpling model<sup>3,4</sup>

$$\dot{\phi}^2 \ll V(\phi), \quad (34)$$

$$\ddot{\phi} \ll D_t H \dot{\phi}, \quad (35)$$

$$-\dot{H} \ll H^2 \quad (36)$$

and

$$\dot{D}_t \left( \ln \frac{a}{a_0} + \frac{d \ln V_{D_t}}{d D_t} \right) \ll D_t H, \quad (37)$$

Eqs. (31) and (32) can be rewritten for a flat decrumppling model, i.e.  $k = 0$ , in the simpler set

$$H^2 \simeq \frac{16\pi V(\phi)}{D_t(D_t - 1)M_P^2}, \quad (38)$$

$$D_t H \dot{\phi} \simeq -V'(\phi). \quad (39)$$

Note that the slow-roll condition (37) has not been considered in Refs. [3,4]. The validity of this condition is obvious by regarding Eq. (11). Substituting (11) in (37), dynamics of the spatial dimension is given by<sup>3,4</sup>

$$\dot{D}_t^2 \simeq \frac{16\pi D_t^3 V(\phi)}{C^2(D_t - 1)M_P^2}. \quad (40)$$

During inflation,  $H$  is slowly varying in the sense that its change per Hubble time  $\epsilon \equiv -\dot{H}/H^2$  is less than one. The slow-roll condition  $|\eta| \ll 1$  is actually a consequence of the condition  $\epsilon \ll 1$  plus the slow-roll approximation  $D_t H \dot{\phi} \simeq -V'(\phi)$ . Differentiating (39) one finds

$$\frac{\ddot{\phi}}{H\dot{\phi}} = \epsilon - \eta + \frac{D_t}{C}, \quad (41)$$

where the slow-roll parameters in decrumppling model are defined by

$$\epsilon \equiv \frac{(D_t - 1)M_P^2}{32\pi} \left( \frac{V'}{V} \right)^2, \quad (42)$$

$$\eta \equiv \frac{(D_t - 1)M_P^2}{16\pi} \left( \frac{V''}{V} \right). \quad (43)$$

It should be emphasized that the slow-roll parameters in decrumppling model as presented in Refs. [3,4] are different from those given in (42) and (43). This difference is due to the slow-roll condition (37) which has not been considered in Refs. [3,4]. Furthermore, in the constant  $D$ -space, the slow-roll parameters (42) and (43) are also valid by substituting  $D_t$  by  $D$ , see Refs. [3,4].

#### 4.2. Explicit formulae for running in decrumppling or TVSD inflation

The amplitudes of scalar and tensor perturbations generated in decrumppling or TVSD inflation can be expressed by<sup>11,14</sup>

$$A_S^2 = \left( \frac{H}{2\pi} \right)^2 \left( \frac{H}{\dot{\phi}} \right)^2, \quad (44)$$

and

$$A_T^2 = \frac{8\pi}{M_P^2(D_t - 1)} \left( \frac{H}{2\pi} \right)^2. \quad (45)$$

These amplitudes are equal to  $\frac{25}{4}$  times the amplitudes as given in Ref. [14]. The amplitudes of scalar and tensor perturbations generated in decrumpling inflation can be determined by substituting (38) and (39) in (44) and (45)

$$A_S^2 = \frac{9216\pi V^3}{D_t^3(D_t - 1)^3 M_P^6 V'^2}, \quad (46)$$

$$A_T^2 = \frac{32V}{D_t(D_t - 1)^2 M_P^4}. \quad (47)$$

These expressions are evaluated at the horizon crossing time when  $k = aH$ . Since the value of Hubble constant does not change too much during inflationary epoch, we can obtain  $dk = Hda$  and  $d \ln k = Hdt = da/a$ . Using the slow-roll condition in decrumpling inflation

$$\frac{d}{d \ln k} = -\frac{V'}{D_t H^2} \frac{d}{d\phi}, \quad (48)$$

and also the dimensional constraint (10) of the model we have

$$\frac{dD_t}{da} = -\frac{D_t^2}{Ca}, \quad (49)$$

$$\frac{dD_t}{d \ln k} = -\frac{D_t^2}{C}, \quad (50)$$

$$\frac{dD_t}{d\phi} = \frac{D_t^3 H^2}{V' C}. \quad (51)$$

After a lengthy but straightforward calculation by using (38), (39), (42), (43) and (46)-(51) we find

$$n_S - 1 \equiv \frac{d \ln A_S^2}{d \ln k} = -6\epsilon + 2\eta + \frac{3D_t(2D_t - 1)}{C(D_t - 1)}, \quad (52)$$

$$n_T \equiv \frac{d \ln A_T^2}{d \ln k} = -2\epsilon + \frac{D_t(3D_t - 1)}{(D_t - 1)C}. \quad (53)$$

where  $n_S$  and  $n_T$  are the spectral indices of scalar and tensor perturbations, respectively. If  $n_S$  and  $n_T$  are expressed as a function of e-folding  $\mathcal{N}$ , one can use the fact that  $\frac{d}{d \ln k} = -\frac{d}{d\mathcal{N}}$  to obtain the desired derivatives even more easily.

To calculate the running of the scalar spectral index, we use the following expressions

$$\frac{d\epsilon}{d \ln k} = -\frac{D_t^2}{C(D_t - 1)}\epsilon - 2\epsilon\eta + 4\epsilon^2, \quad (54)$$

$$\frac{d\eta}{d \ln k} = -\frac{D_t^2}{C(D_t - 1)}\eta + 2\epsilon\eta - \xi, \quad (55)$$

where the third slow-roll parameter is defined by

$$\xi \equiv \frac{(D_t - 1)^2 M_P^4}{(16\pi)^2} \left( \frac{V' V'''}{V^2} \right). \quad (56)$$

Using above equations, running of the scalar spectral index in decrumpling or TVSD inflation has this explicit expression

$$\frac{dn_S}{d \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi + \frac{6D_t^2}{C(D_t - 1)}\epsilon - \frac{2D_t^2}{C(D_t - 1)}\eta - \frac{3D_t^2(2D_t^2 - 4D_t + 1)}{C^2(D_t - 1)^2}. \quad (57)$$

The standard consistency relation between the tensor to scalar perturbations and the tensor spectral index is

$$R = -8n_T, \quad (58)$$

where  $R$  is the ratio of tensor to scalar perturbations<sup>14</sup>

$$R \equiv 16 \frac{A_T^2}{A_S^2}. \quad (59)$$

This equation can be rewritten in decrumpling or TVSD inflation as

$$R \equiv 16 \frac{A_T^2}{A_S^2} + f\left(\frac{1}{C}\right), \quad (60)$$

where  $f\left(\frac{1}{C}\right)$  is a function of the inverse of universal constant of decrumpling or TVSD model, see Eq.(10). From Eqs.(46), (47) and (60), one can obtain

$$R \equiv \frac{16D_t^2}{9}\epsilon + f\left(\frac{1}{C}\right). \quad (61)$$

The consistency equation in decrumpling or TVSD inflation is

$$R = -\frac{8D_t^2}{9}n_T. \quad (62)$$

In the limit  $D_t = 3$ , Eq.(62) yields Eq.(58). From Eqs.(53) and (62), we have

$$R = \frac{16D_t^2}{9}\epsilon - \frac{8D_t^3(3D_t - 1)}{9(D_t - 1)C}. \quad (63)$$

Comparing this equation with (61) gives

$$f\left(\frac{1}{C}\right) = -\frac{8D_t^3(3D_t - 1)}{9(D_t - 1)C}. \quad (64)$$

Let us now study two inflationary potentials. We first study  $V(\phi) = m^2\phi^2/2$  and then  $V(\phi) = \lambda\phi^4$ .

#### 4.2.1. The first example: $V(\phi) = \frac{1}{2}m^2\phi^2$

For the purposes of illustration, we now consider the potential  $V(\phi) = m^2\phi^2/2$

$$\epsilon = \eta = \frac{(D_t - 1)M_P^2}{8\pi\phi^2}, \quad (65)$$

$$\xi = 0. \quad (66)$$

Using the definition of e-folding in decrumppling inflation<sup>11</sup>

$$\mathcal{N} = -\frac{16\pi}{M_P^2} \int_{\phi}^{\phi_f} \frac{V}{(D_t - 1)V'} d\phi, \quad (67)$$

we have

$$\mathcal{N} = \frac{4\pi}{(D_t - 1)M_P^2} (\phi^2 - \phi_f^2), \quad (68)$$

where  $\phi_f$  is the value of the inflaton field at the end of inflation. Note that in the integral of (67),  $D_t$  is as a function of the inflaton field. To integrate in (67) we take  $D_t$  to be independent on the inflaton field because the relationship between  $D_t$  and  $\phi$  in decrumppling inflation is too weak.<sup>3</sup> For this reason, our approximation is appropriate for integration of Eq. (67).

To obtain e-folding as a function of the inflaton field, we must obtain  $\phi_f$ . From  $\epsilon = \eta = 1$  we get

$$\phi_f = \sqrt{\frac{D_t - 1}{8\pi}} M_P. \quad (69)$$

From (68) and (69), we have

$$\phi^2 = \frac{(D_t - 1)M_P^2}{8\pi} (2\mathcal{N} + 1). \quad (70)$$

So we are led to the spectral index and its running

$$n_S - 1 = -\frac{4}{(2\mathcal{N} + 1)} + \frac{3D_t(2D_t - 1)}{C(D_t - 1)}, \quad (71)$$

$$\frac{dn_S}{d\ln k} = -\frac{8}{(2\mathcal{N} + 1)^2} - \frac{4D_t^2}{C(D_t - 1)(2\mathcal{N} + 1)} - \frac{3D_t^2(2D_t^2 - 4D_t + 1)}{C^2(D_t - 1)^2}. \quad (72)$$

One can obtain the ratio of tensor to scalar perturbations by (63) and (65)

$$R = \frac{2D_t^2(D_t - 1)M_P^2}{9\pi\phi^2} - \frac{8D_t^3(3D_t - 1)}{9(D_t - 1)C}. \quad (73)$$

Substituting (70) in (73) yields

$$R = \frac{16D_t^2}{9(2\mathcal{N} + 1)} - \frac{8D_t^3(3D_t - 1)}{9(D_t - 1)C}. \quad (74)$$

From (71) and (74), one can obtain

$$n_S - 1 = -\frac{9R}{4D_t^2} - \frac{D_t}{C(D_t - 1)}. \quad (75)$$

#### 4.2.2. The second example: $V(\phi) = \lambda\phi^4$

For the second example, we study  $V(\phi) = \lambda\phi^4$ . The slow-roll parameters are

$$\epsilon = \frac{(D_t - 1)M_P^2}{2\pi\phi^2}, \quad (76)$$

$$\eta = \frac{3(D_t - 1)M_P^2}{4\pi\phi^2}, \quad (77)$$

$$\xi = \frac{3(D_t - 1)^2 M_P^4}{8\pi^2\phi^4}. \quad (78)$$

Using the definition of e-folding in inflation, we have

$$\mathcal{N} = \frac{2\pi}{M_P^2(D_t - 1)} (\phi^2 - \phi_f^2). \quad (79)$$

To obtain the e-folding number as a function of the inflaton field, we must obtain the value of the inflaton field at the end of inflation. From  $\epsilon = 1$  we get

$$\phi_f = \sqrt{\frac{D_t - 1}{2\pi}} M_P. \quad (80)$$

Assuming  $\eta = 1$  and  $\xi = 1$ , we obtain

$$\phi_f = \sqrt{\frac{3(D_t - 1)}{4\pi}} M_P \quad (81)$$

and

$$\phi_f = \left( \frac{3(D_t - 1)^2}{8\pi^2} \right)^{1/4} M_P, \quad (82)$$

respectively. These values of  $\phi_f$  based on the condition  $\eta = 1$  and  $\xi = 1$  are larger than  $\phi_f$  arisen from  $\epsilon = 1$ . We here take the condition  $\epsilon = 1$  by itself is a true condition to obtain  $\phi_f$ . From (79) and (80) we have

$$\phi^2 = \frac{(D_t - 1)(\mathcal{N} + 1)M_P^2}{2\pi}. \quad (83)$$

So we are led to the spectral index and its running

$$n_S - 1 = -\frac{3}{(\mathcal{N} + 1)} + \frac{3D_t(2D_t - 1)}{C(D_t - 1)}, \quad (84)$$

$$\frac{dn_S}{d\ln k} = -\frac{3}{(\mathcal{N} + 1)^2} + \frac{3D_t^2}{C(D_t - 1)(\mathcal{N} + 1)} - \frac{3D_t^2(2D_t^2 - 4D_t + 1)}{C^2(D_t - 1)^2}. \quad (85)$$

One can obtain the ratio of tensor to scalar perturbations by (63) and (76)

$$R = \frac{8D_t^2(D_t - 1)M_P^2}{9\pi\phi^2} - \frac{8D_t^3(3D_t - 1)}{9(D_t - 1)C}. \quad (86)$$

Substituting (83) in (86) yields

$$R = \frac{16D_t^2}{9(\mathcal{N} + 1)} - \frac{8D_t^3(3D_t - 1)}{9(D_t - 1)C}. \quad (87)$$

From (84) and (87), one can obtain

$$n_S - 1 = -\frac{27R}{16D_t^2} + \frac{3D_t}{2C}. \quad (88)$$

#### 4.3. Numerical calculations

The WMAP team<sup>8</sup> carried out the likelihood analysis by varying the four quantities  $A_S$ ,  $R$ ,  $n_S$  and  $dn_S/d\ln k$ . The quantities  $n_T$  and  $dn_T/d\ln k$  are related to those by consistency equation, and  $dn_T/d\ln k$  has anyway always been ignored so far in parameter fits as its cosmological consequences are too subtle for current or near-future data to detect. Thus, we ignore it and use four observables ( $A$ ,  $R$ ,  $n_S$ ,  $dn_S/d\ln k$ ) as free parameters.

As shown in (46), the value of  $A_S$  in decrumpling model is independent of the inverse of universal constant,  $\frac{1}{C}$ , of the model. This means that the value of  $A_S$  in the standard inflation in  $D$ -constant spatial dimension is equal to the value of  $A_S$  in decrumpling model. We therefore conclude that the characteristic of time variability of the space dimension in decrumpling inflation does not change the value of  $A_S$ .

##### 4.3.1. The first potential: $V(\phi) = m^2\phi^2/2$

Let us calculate the value of  $n_S - 1$  and  $dn_S/d\ln k$  for the potential  $m^2\phi^2/2$ . Substituting  $D_t = 3$  for the present value of the space dimension in Eqs. (71) and (72)

$$n_S - 1 = -\frac{4}{(2\mathcal{N} + 1)} + \frac{45}{2C}, \quad (89)$$

$$\frac{dn_S}{d\ln k} = -\frac{8}{(2\mathcal{N} + 1)^2} - \frac{18}{C(2\mathcal{N} + 1)} - \frac{189}{4C^2}. \quad (90)$$

From Eqs. (89) and (90), one can obtain for  $C = 599.57$  corresponding to  $D_P = 10$ , varying  $(n_S - 1)$  from  $-0.00207$  to  $0.00916$  and  $\mathcal{N}$  from  $50$  to  $70$ ,  $dn_S/d\ln k$  varies from about  $-0.00121$  to almost  $-0.00075$ . In this case, the value of  $n_S - 1$  vanishes within about  $\mathcal{N} = 52.8$  e-folding. In other words, the sign of  $n_S - 1$  within about  $\mathcal{N} = 52.8$  changes from red ( $n_S < 1$ ) to blue ( $n_S > 1$ ) tilt.

We obtain for  $C = 1678.8$  corresponding to  $D_P = 4$ , varying  $(n_S - 1)$  from  $-0.0262$  to  $-0.0150$  and  $\mathcal{N}$  from  $50$  to  $70$ ,  $dn_S/d\ln k$  varies from about  $-0.0009$  to almost  $-0.0005$ .

For  $C \rightarrow +\infty$  corresponding to the constant space dimension and  $D_P = 3$ , varying  $(n_S - 1)$  from  $-0.03960$  to  $-0.02836$  and  $\mathcal{N}$  from  $50$  to  $70$ ,  $dn_S/d\ln k$  varies from about  $-0.00078$  to almost  $-0.00040$ .

It is important to study the ratio of tensor to scalar perturbations as a function of  $n_S$  to check whether or not this ratio lies within the observational sigma bounds.<sup>14</sup> From (75), one can obtain in three-space dimension

$$n_S - 1 = -\frac{R}{4} - \frac{3}{2C}. \quad (91)$$



To plot  $n_S$  and  $R$  together with the  $2D$  posterior constraints in the  $n_S - R$  plane, we need to run the CAMB program coupled to the CosmoMc (Cosmological Monte Carlo) code, for more details see Ref.[14] and references therein. We will study this problem in our future works.

#### 4.3.2. The second potential: $V(\phi) = \lambda\phi^4$

We now calculate the value of  $n_S - 1$  and  $dn_S/d\ln k$  for the potential  $\lambda\phi^4$ . Substituting  $D_t = 3$  for the present value of the space dimension in Eqs. (84) and (85)

$$n_S - 1 = -\frac{3}{(\mathcal{N} + 1)} + \frac{45}{2C}, \quad (92)$$

$$\frac{dn_S}{d\ln k} = -\frac{3}{(\mathcal{N} + 1)^2} + \frac{27}{2C(\mathcal{N} + 1)} - \frac{189}{4C^2}. \quad (93)$$

From Eqs. (92) and (93), one can obtain for  $C = 599.57$  corresponding to  $D_P = 10$ , varying  $(n_S - 1)$  from  $-0.02130$  to  $-0.00473$  and  $\mathcal{N}$  from  $50$  to  $70$ ,  $dn_S/d\ln k$  varies from about  $-0.00084$  to almost  $-0.00041$ .

We obtain for  $C = 1678.8$  corresponding to  $D_P = 4$ , varying  $(n_S - 1)$  from  $-0.04542$  to  $-0.02885$  and  $\mathcal{N}$  from  $50$  to  $70$ ,  $dn_S/d\ln k$  varies from about  $-0.00101$  to almost  $-0.00050$ .

For  $C \rightarrow +\infty$  corresponding to the constant space dimension and  $D_P = 3$ , varying  $(n_S - 1)$  from  $-0.05882$  to  $-0.04225$  and  $\mathcal{N}$  from  $50$  to  $70$ ,  $dn_S/d\ln k$  varies from about  $-0.00115$  to almost  $-0.00059$ .

It is important to study the ratio of tensor to scalar perturbations as a function of  $n_S$  to check whether or not this ratio lies within the observational sigma bounds.<sup>14</sup> From (88), one can obtain in three-space dimension

$$n_S - 1 = -\frac{3R}{16} + \frac{9}{2C}. \quad (94)$$

To plot  $n_S$  and  $R$  together with the  $2D$  posterior constraints in the  $n_S - R$  plane, we need to run the CAMB program coupled to the CosmoMc (Cosmological Monte Carlo) code, for more details see Ref.[14] and references therein. We will study this problem in our future works.

## 5. Conclusions

In previous our studies about time variable spatial dimension model<sup>1</sup> we showed that based on observational bounds on the present-day variation of Newton's constant, one would have to conclude that the spatial dimension of the Universe when the Universe was at the Planck length to be less than or equal to  $3.09$ . In [1] we concluded that if the dimension of space when the Universe was at the Planck scale is constrained to be fractional and very close to  $3$ , then the whole edifice of TVSD model loses credibility.<sup>1</sup>

In this paper, we have studied the effects of time variability of the spatial dimension on the time evolution of inflaton field, space dimension and scale factor for the potential  $m^2\phi^2/2$ . We have also obtained general and explicit formulae for the scalar spectral index and its running in TVSD or decrumpling model and then applied them to two classes of examples of the inflaton potential. The correction terms due to time variability of space dimension depend on the e-folding number and the universal constant of TVSD model which is  $C$ . The numerical calculations for the spectral index and its running have been done for two classes of examples.

We have also discussed on dynamical solutions of inflaton field, scale factor and space dimension in TVSD or decrumpling chaotic inflation model. The outline of results has been shown in Figs. 1, 2 and 3. Fig.1 compares inflaton field within the framework of TVSD model and the constant three-space chaotic inflation. It is seen time variability of space dimension causes that inflaton field evolves slowly. This behavior of the scalar field can be seen by Eq. (15) where the effect of time variable dimension is like decreasing friction term in this equation, or subsequently causes that the scalar field changes slowly. In Fig. 2 the evolution of space dimension has been obtained. It is seen from Eq. (11) that temporal rate of space dimension is proportional to the square of space dimension. So during the inflationary epoch the Universe with  $D_P = 10$  loses 6 dimensions in comparison to  $D_P = 4$  which loses less than one. In other words,  $\Delta D_{\text{inflation}}^{D_P=10} = 6$  and  $\Delta D_{\text{inflation}}^{D_P=4} < 1$ . From Fig.3 it is seen that dynamical character of the space dimension increases the e-folding number.<sup>4</sup> Starting an Universe with higher dimensions at the Planck epoch, it leaves inflationary phase later.<sup>3</sup>

The final point that must be emphasized is about decrumpling model in cosmology. The original motivation of this model presented in the pioneer paper<sup>12</sup> was based on an *ad hoc* assumption inspired from polymer physics. It is quite possible that this part of decrumpling model should be revised. However, just how this should be done is far from obvious. The progress in decrumpling model can only be made if there is a breakthrough in terms of finding a natural mechanism for varying the spatial dimension in some alternative fashion to that which we have considered.

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